Mathematics: analysis and approaches
Higher level
Paper 1

Topic: Derivatives

Candidate name

75 minutes

Instructions to candidates

- Write your name in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Answer all questions.
- Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[53 marks]**.

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Using the first principle, show that the derivative of $(x + 1)^3$ is $3(x + 1)^2$.

2. [Maximum mark: 6]

Find the limit: $\lim_{x \to 0} \frac{sec^4 x - cos^2 x}{x^4 - x^2}$.

3. [Maximum mark: 8]

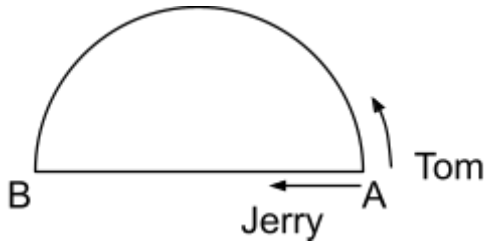
Consider the curve with the equation $x^3 + 4y^3 = xy$.

(a) Find an expression for $\frac{dy}{dx}$. [3]

(b) Find the coordinates of all points where the tangent to the curve is vertical. [5]

4. [Maximum mark: 8]

Tom and Jerry travels from point A to point B along two different paths: Tom travels along the semi-circle with radius of 1 km at the speed of 1 km/h, and Jerry travels along the diameter AB at the speed of $\frac{1}{\pi}$ km/h.



Find the rate of change of the distance between Tom and Jerry at the time when Jerry travels half way through.

5. [Maximum mark: 7]

Consider the function $f(x) = \sqrt{x^2 ln(x) + 4 - x^2}$, where $x \in \mathbb{R}, x > 0$.

(a) Show that the distance , l, between the origin and any point on the graph is given by $l = \sqrt{x^2 ln(x) + 4}$. [1]

(b) Hence, find the x-coordinate of the point on the graph of f which is the closest to the origin. [6]

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

6. [Maximum mark: 19]

Consider the curve C defined by the equation $e^{x+y} - 1 = x^2 + y^2$.

(a) Show that $\frac{dy}{dx} = \frac{2x - e^{x+y}}{e^{x+y} - 2y}$. [5]

(b) Show that there is no point on the graph where the tangent is horizontal. [7]

(c) Show that the graph of C is symmetric about the line $y = x$. [3]

(d) Find the coordinates of the point on the curve C where the tangent has a gradient of -1. [4]